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BOUNDED ERROR ADAPTIVE CONTROL, PART II.(U)
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6) BOUNDED ERROR ADAPTIVE CONTROL.

Part II.

10) Kumpati S./Narendra ~~and~~ Benjamin B./Peterson

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Bounded Error Adaptive Control

Part II

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Abstract

The nonlinear adaptive algorithm suggested in Part I is extended to more general adaptive control problems with external disturbances. Conditions for the boundedness of all the signals of the overall adaptive system are derived. The prior information needed to determine the size of the dead zone in the adaptive law is briefly discussed.

## I. Introduction:

The global stability of adaptive control schemes for deterministic systems was recently established by several authors [1-4]. This report examines the effect of external disturbances on the stability of such schemes and proposes a nonlinear adaptive law which, under certain conditions, assures the boundedness of all signals in the overall system. These conditions may be stated in terms of the prior information needed regarding the plant transfer function and the external disturbance. In particular, it is assumed that the order n, relative degree $n^*$ and high frequency gain $k_p$ of the plant transfer function $W_p(s)$ are known and that the zeros of $W_p(s)$ lie in the open left half of the complex plane. Further it is also assumed that a bound on the disturbance is known and that adequate prior information regarding the plant transfer function is available to determine a bound on the effect of the disturbance at the output.

In part I of this report [5] an error model containing an output disturbance was analyzed in detail. When the input to the error model is uniformly bounded sufficient conditions were derived for the stability as well as instability of the system. Such an error model finds direct application in adaptive observers where the input to the error model can be assumed to be bounded without loss of generality. In the control problem, however, where such an assumption cannot be made, it was shown that the parameter error can grow in an unbounded fashion.

The principal difficulty in the adaptive control problem discussed arises while determining the direction along which the control parameter vector is to be adjusted when the output error is small. The nonlinear adaptive law suggested in part I uses a dead zone so that adaptation takes place only when the output error is large and a decrease in the magnitude of the parameter error vector is assured. This approach is extended in this report to more general adaptive control problems. In section II the case of a plant in which the entire state vector can

be measured in the presence of disturbance is considered and provides the motivation for the subsequent analysis. The principal result of the report, concerned with sufficient conditions for the stable adaptive control of a single-input single-output linear time-invariant plant with output disturbance is contained in section III.

When the amplitude of the output error is smaller than the size of a dead zone, adaptation ceases and the overall system is linear and time-invariant. When the output error exceeds the size of the dead zone, adaptive adjustment of the control parameters takes place and the overall system is nonlinear and time-varying. In view of these two modes of behavior exhibited by the adaptive loop, the proof of stability in [1] for the disturbance free case cannot be directly extended to this problem. The proof of stability in section III is by contradiction. It is shown that the augmented error must lie outside the dead zone for an infinite time if the signals within the system are to become unbounded and this in turn results in the contradiction.

Concurrent work at Yale [6], which complements the results presented here, considers the same adaptive control problem of a linear time-invariant plant under similar assumptions. The adaptive law, in this case, is similar to that used in the disturbance free case when $\|\theta(t)\| \leq \|\theta\|_{max}$ and is modified only when $\|\theta(t)\| > \|\theta\|_{max}$. Using such an adaptive law it is shown that all the parameters and signals of the feedback system remain bounded. However, due to the presence of the disturbance, the parameter vector $\theta(t)$ continues to be adjusted for all time. When the external disturbance is not present (or tends to zero) the approach yields zero output error in the limit. In contrast to the above scheme, the adaptive law in the present case is modified (by the dead zone) only when the output error is small. This results in the parameter vector $\theta(t)$ tending to a constant value in most cases and adaptation ceasing altogether as $t \to \infty$. The presence of the dead

zone in the adaptive law however precludes the possibility of the output error becoming zero in the limit even when the external disturbance is not present. Combining the ideas contained in [6] and the present report appears to have considerable potential for practical applications.

## II. Adaptive Control Using the State of the Plant:

To complement the approach used in section III for the adaptive control of a single-input single-output plant we shall consider here the relatively simple case of a plant whose state variables can be measured in the presence of a vector additive disturbance.

The plant and model are assumed to be described by the equations[*]

$$\left.\begin{aligned} \dot{x}_p &= A_p x_p + bu \\ \bar{x}_p &= x_p + v \end{aligned}\right\} \text{Plant} \qquad (1)$$

$$\dot{x}_m = A_m x_m + br \qquad \text{Model} \qquad (2)$$

where v is a vector of uniformly continuous bounded disturbances, $(A_p, b)$ is controllable, $A_m$ is a known constant stable matrix, b a known constant vector, $A_p$ is a constant matrix with unknown elements, and r is a piecewise continuous bounded reference input. It is further assumed that a vector $k^*$ exists such that

$$[A_p + bk^{*T}] = A_m \qquad (3)$$

The structure of the adaptive controller is shown in Figure 1.

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* In sections II and III the argument 't' is omitted in all the equations except when necessary for clarity.

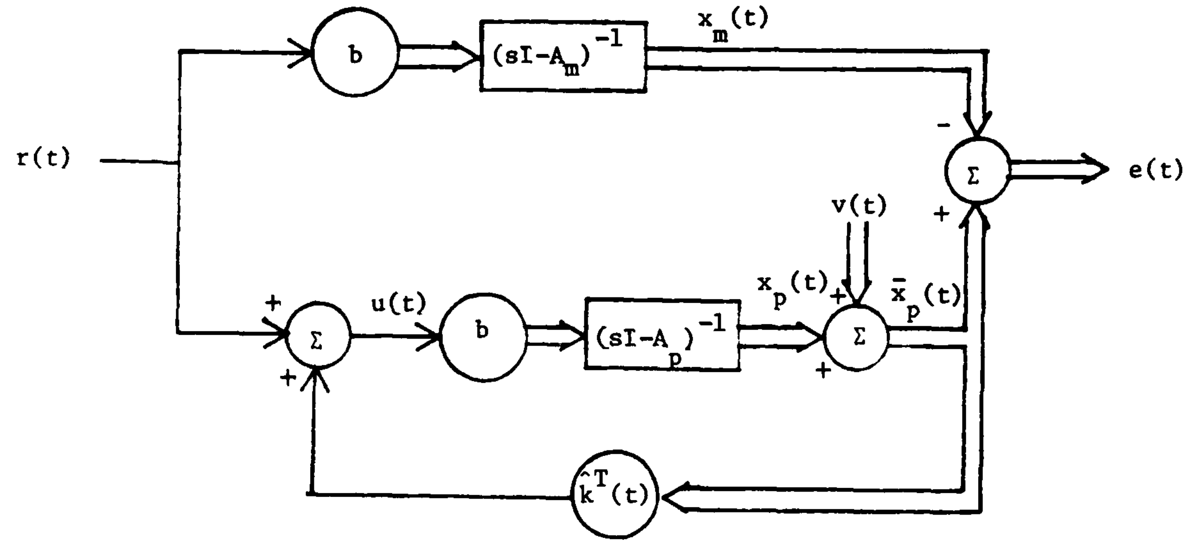



Figure 1

The basic idea of the adaptive controller is to determine a control input u such that the error between plant and model outputs as well as all signals within the system remain uniformly bounded. Due to the disturbance it is not possible to insure that $\lim_{t\to\infty} \|e(t)\| = 0$ as in the noise free case [7] but only that it will remain bounded. For purposes of analysis v(t) will be replaced by the equivalent vector input disturbance and $\bar{x}_p$ will be the state vector which can now be measured exactly.

$$\dot{\bar{x}}_p = \dot{x}_p + \dot{v} \tag{4}$$

$$= A_p x_p + bu + \dot{v}$$

$$= A_p \bar{x}_p + bu + w \tag{5}$$

where $w \triangleq \dot{v} - A_p v$ is the equivalent vector disturbance at the input. If the control input u in equation (5) is $u = r + \hat{k}^T \bar{x}_p$ we have

$$\dot{\bar{x}}_p = [A_p + b\hat{k}^T]\bar{x}_p + br + w$$

Defining the measured state vector error as

$$e \triangleq \bar{x}_p - x_m$$

and the parameter error vector as

$$\phi(t) \triangleq \hat{k}(t) - k^*$$

the error model shown in Figure 2 is described by

$$\dot{e} = A_m e + b\phi^T \bar{x}_p + w \tag{6}$$

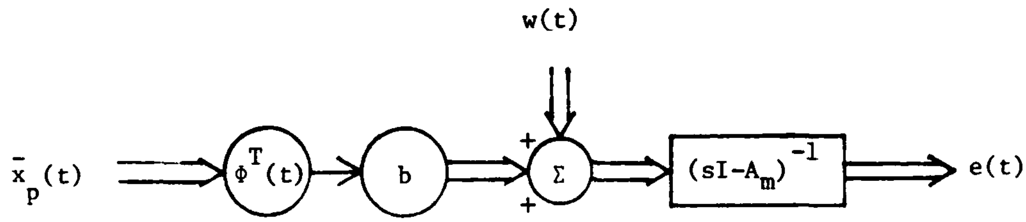



Figure 2

a) <u>The Adaptive Law</u>:

In the absence of the disturbance v it is well-known [7] that the adaptive law

$$\dot{\phi} = -e^T Pbx_p \tag{7}$$

where $P = P^T > 0$, $A_m^T P + PA_m = -Q < 0$ and $b^T P(sI-A_m)^{-1}b$ is a strictly positive real transfer function, results in a globally stable system with $\lim_{t\to\infty} e(t) = 0$.

Following the results of Part I [5] the adaptive law is modified as follows.

$$\begin{aligned} \dot{\phi} &= 0 & \quad e^T Pe &\le E_0 \\ \dot{\phi} &= -e^T Pb\bar{x}_p & \quad e^T Pe &> E_0 \end{aligned} \tag{8}$$

where $E_0$ defines the size of a dead zone. In general $E_0$ can be set larger than some minimum value for stability as shown in the following section. A larger $E_0$ implies that a larger output error vector is tolerated and adaptation takes place over a shorter total time.

b) Analysis of Stability:

If $E_0$ is chosen such that

$$E_0 > \frac{4\lambda_1^3 w_0^2}{\lambda_2^2} + \delta \qquad \delta > 0 \tag{9}$$

where $\lambda_1$ = maximum eigenvalue of P

$\lambda_2$ = minimum eigenvalue of Q

and $w_0$ is the uniform bound on $\|w(t)\|$.

Let $\Omega_1,\Omega_2$ be defined as $\Omega_1 \triangleq \{t|e^TPe \leq E_0\}$, $\Omega_2 \triangleq \{t|e^TPe > E_0\}$ so that $\Omega_1,\Omega_2$ is a partitioning of $R^+$.

Defining the function $V(e,\phi)$ as:

$$\begin{aligned} V(e,\phi) &= E_0 + \phi^T\phi \qquad && e^TPe \leq E_0 \quad (t \in \Omega_1) \\ &= e^TPe + \phi^T\phi && e^TPe > E_0 \quad (t \in \Omega_2) \end{aligned}$$

$V(e,\phi)$ has the following properties.

i. $V \geq E_0 > 0$ (10)

ii. V is continuous in the 2n dimensional $(e,\phi)$ space.

iii. $\dot{V} = 0 \qquad t \in \Omega_1$

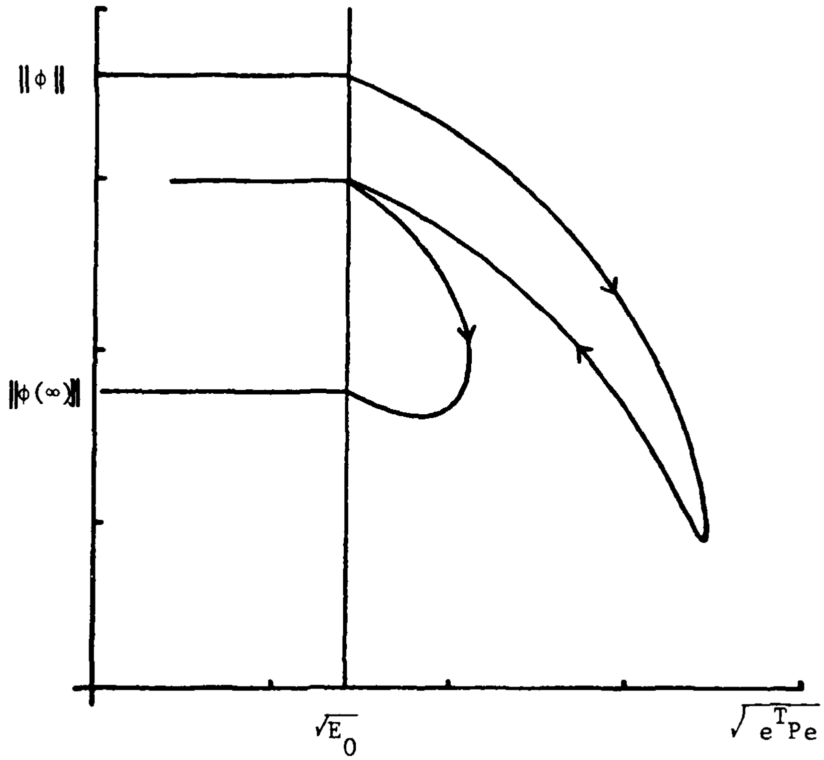
$\dot{V} = -e^TQe + 2e^TPw < -\varepsilon(\delta) < 0 \qquad t \in \Omega_2$

iv. $V \geq \lambda_3\|e\|^2 + \|\phi\|^2$

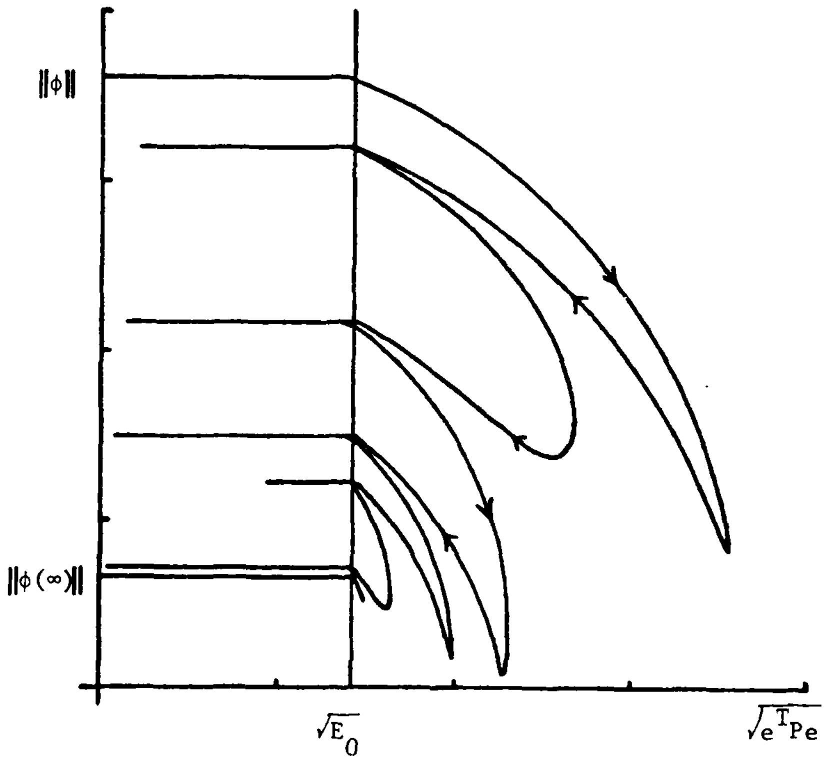
where $\lambda_3 > 0$ is the minimum eigenvalue of P.

Assuming that $\|e(t_0)\|$ and $\|\phi(t_0)\|$ are bounded, conditions (i)-(iv) imply that $\|e(t)\|$ and $\|\phi(t)\|$ are bounded for all $t \in [t_0,\infty)$. Further since $\dot{V} < -\varepsilon(\delta) < 0$ for $t \in \Omega_2$ the total time during which adaptation takes place, defined by $\mu[\Omega_2]$, is finite.

A computer simulation illustrating the nature of the trajectories in a two-dimensional space defined by $\sqrt{e^TPe}$ and $\|\phi\|$ is shown in Figure 3. For $t \in \Omega_1$, $\phi$ is a constant and the trajectory is a horizontal line. Let the trajectory be on




3a. r = 16 cos 5t




3b. r = 40 cos 5t

Figure 3

the boundary of $\Omega_1$ for some time $t_2$ i.e. $e^T(t_2)Pe(t_2) = E_0$. Further, let $(t_2,t_3) \subset \Omega_2$. If $\phi(t_2)$ is finite, since $\dot{V} \lesssim \varepsilon < 0$, the length of the interval $(t_2 t_3)$ must be finite $\left(t_3-t_2 \leq \frac{\phi^T(t_3)\phi(t_2)}{\varepsilon}\right)$ and there exists a finite time $t_4$ at which the trajectory lies on the boundary of the dead zone i.e. $t_4 \in \Omega_1$. Since $e^T(t_2)Pe(t_2) = e^T(t_4)Pe(t_4) = E_0$ and $\dot{V} < \varepsilon < 0$ it follows that $\|\phi(t_4)\| < \|\phi(t_2)\|$ and hence the norm of the parameter error decreases a finite amount every time the trajectory leaves the dead zone and re-enters. In the limit $\phi$ is a constant and the trajectory is such that $t \in \Omega_1$ for all $t \geq T$, $T \in R^+$.

Example 1: In the adaptive system simulated, [Figure 3] the plant and model parameters are given by:

$$A_p = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \qquad A_m = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and } w(t) = \begin{bmatrix} \sin 5t \\ \sin 5t \end{bmatrix}$$

The desired feedback control parameter vector is

$$k^* = \begin{bmatrix} -1 \\ -2 \end{bmatrix} \quad \text{and } Q = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}.$$

The dead zone was calculated using (9) as $\frac{4\lambda_1^3 w_0^2}{\lambda_2^2} = 2.$

Figure 3a shows the trajectories for a reference input $r = 16 \cos 5t$ and 3b for a reference input $r = 40 \cos 5t$. In both cases the parameter error decreases when the error lies outside the dead zone and in (3b) the final parameter error is smaller.

The ideas in this section are also applicable to adaptive control of minimum phase single-input single-output plants of relative degree $n^* = 1$ where the plant output is measured in the presence of a bounded and uniformly continuous disturbance.

If the relative degree is greater than one, or if the disturbance cannot be assumed to be uniformly continuous, a more complex controller structure is required. The stability analysis for this case is the main result of this report and is presented in the following section

## III. Adaptive Control of a Single-Input Single-Output Plant:

### a) The Plant:

It is assumed that the plant to be controlled can be modeled by the linear time-invariant differential equations

$$\begin{aligned} \dot{x}_p &= A_p x_p + b_p u \\ y_p &= c^T x_p + v_1 \end{aligned} \tag{12}$$

where $x_p$ is the state of the plant, $v_1$ is a bounded output disturbance and $y_p$ the measured plant output. The transfer function of the plant is

$$c^T[sI-A_p]^{-1}b_p \triangleq W_p(s) = k_p \frac{N_p(s)}{D_p(s)} \tag{13}$$

It is assumed that

(i) $D_p(s)$ is a monic polynomial of degree n.

(ii) $N_p(s)$ is a monic Hurwitz polynomial of degree $m(\leq n-1)$.

(iii) the output disturbance $v_1$ is uniformly bounded and the bound on $v_1$ $(= v_{1max})$ is known.

(iv) $n$, $n^* \triangleq n - m$, the gain $k_p$ are known.

### b) The Model:

A reference model is described by the vector equations

$$\begin{aligned} \dot{x}_m &= A_m x_m + b_m r \\ y_m &= c_m^T x_m \end{aligned} \tag{14}$$

where

(i) the transfer function $W_m(s) = \frac{k_m}{D_m(s)}$

and $D_m(s)$ is a monic Hurwitz polynomial of degree $n^*$.

(ii) r is a piecewise continuous, uniformly bounded reference input.

The control problem then is to determine a suitable bounded control input u(t) such that $e(t) \triangleq y_p(t) - y_m(t)$ remains bounded for all $t \geq t_0$. In the following analysis we assume that $k_p = k_m = 1$ for ease of exposition.

c) Structure of the Adaptive Controller:

The controller structure has the same form as that used in the disturbance free case [1] and may be described as follows:

Using the plant input u and measured output $y_p$, a (2n-1) dimensional auxiliary vector $\omega$ is generated as

$$\begin{aligned} \dot{\omega}_{(1)} &= F\omega_{(1)} + gu \\ \dot{\omega}_{(2)} &= F\omega_{(2)} + gy_p \end{aligned} \qquad \text{Auxiliary Signal Generator} \qquad (15)$$

where F is a stable matrix, (F,g) is a controllable pair and $\omega^T = [\omega^T_{(1)}, \omega^T_{(2)}, y_p]$.

The input to the plant is given by

$$u = r + \theta^T \omega \qquad (16)$$

where $\theta^T(t)$ is a (2n-1) dimensional control parameter vector. $[\theta^T_{(1)}, \theta^T_{(2)}, \theta_{2n-1}]$ Since a constant parameter vector $\theta^*$ exists such that for $\theta(t) \equiv \theta^*$ the transfer function of the plant together with the controller matches that of the model [8], the model output and the measured plant output may be expressed as

$$\begin{aligned} W_m(s) r &= y_m \\ W_m(s)[r + \phi^T \omega] + v &= y_p \end{aligned} \qquad (17)$$

where $\theta(t) - \theta^* \triangleq \phi$ and v is the effect of the disturbance $v_1$ at the output.

(i.e. v is the measured output error if $\theta \equiv \theta^*$). From (17) the measured error e(t) can be expressed as

$$e \triangleq y_p - y_m = W_m(s)\phi^T\omega + v \tag{18}$$

An auxiliary error signal $\bar{\varepsilon}(t)$ is defined by

$$\begin{aligned}\bar{\varepsilon} &= \theta^T W_m(s)\omega - W_m(s)\theta^T\omega \\ &= \phi^T W_m(s)\omega - W_m(s)\phi^T\omega\end{aligned} \tag{19}$$

and the augmented error $\varepsilon(t)$, used in the adaptive law is given by

$$\varepsilon(t) = e(t) + \bar{\varepsilon}(t). \tag{20}$$

Equations (15),(16),(19), and (20) define the controller structure. The plant together with the controller is shown in Figure 4.

d) <u>The Adaptive Law</u>:

From equations (18-20) the augmented error $\varepsilon(t)$ may be expressed as

$$\varepsilon = \phi^T W_m(s)\omega + v = \phi^T\zeta + v \tag{21}$$

where $\zeta \triangleq W_m(s)\omega$.

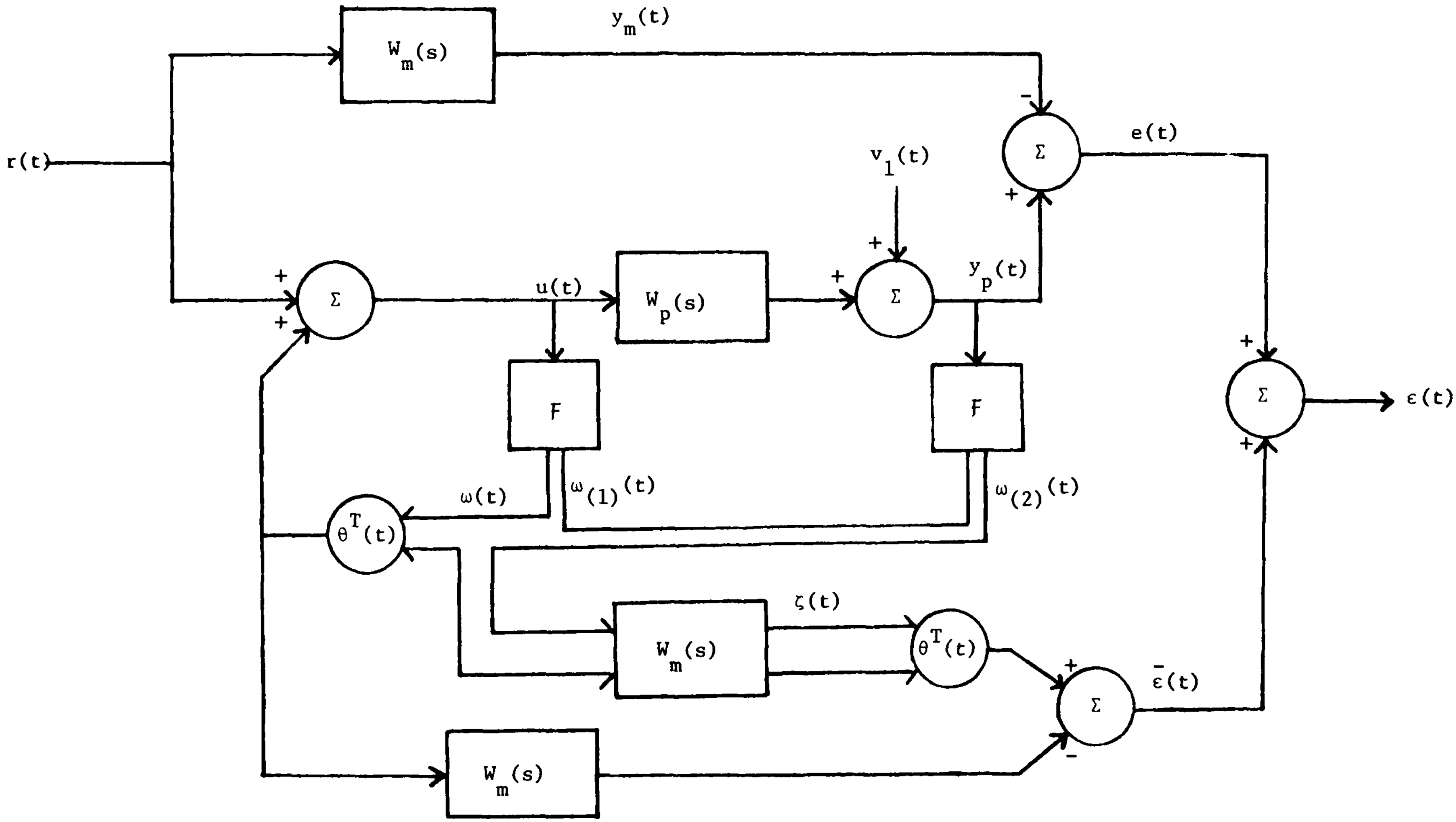
Equation (21) is the principal error equation used for determining the adaptive law. In the disturbance free case (i.e. $v(t) \equiv 0$), the adaptive law

$$\dot{\phi}(t) = -\frac{\Gamma\varepsilon(t)\zeta(t)}{1 + \zeta^T(t)\Gamma\zeta(t)} \qquad \Gamma = \Gamma^T > 0 \tag{22}$$

was shown to result in $\lim_{t\to\infty} e(t) = 0$ while the parameters and the signals of the adaptive loop remain bounded.

Following the procedure outlined in Part I as well as the previous section of this report we modify this adaptive law to take into account the effect of the disturbance. From equation (21) we have

$$\varepsilon = \phi^T\zeta + v$$



r(t)
$W_m(s)$
$y_m(t)$
Σ
e(t)
$v_1(t)$
u(t)
$W_p(s)$
$y_p(t)$
F
F
Σ
ε(t)
ω(t)
$\omega_{(1)}(t)$
$\omega_{(2)}(t)$
$\theta^T(t)$
ζ(t)
$W_m(s)$
$\theta^T(t)$
$\bar{\varepsilon}(t)$
$W_m(s)$


Figure 4

The additive disturbance v is related to $v_1$ and the unknown plant parameters. It can be shown that if $v_1$ is uniformly bounded and $W_m(s)$ and F are stable, v is uniformly bounded. For the following analysis we shall assume that a bound $v_0$ of $|v|$ can be determined even though the plant parameters are unknown. This aspect of the problem is considered further in section IV.

For the error equation (21) the nonlinear adaptive law

$$\begin{aligned} \dot{\phi} &= -\frac{\Gamma\varepsilon\zeta}{1+\zeta^T\Gamma\zeta} & |\varepsilon(t)| &> v_0+\delta \\ &= 0 & |\varepsilon(t)| &\le v_0+\delta \end{aligned} \tag{23}$$

is used, where $|v(t)| \le v_0$ and $\delta$ is an arbitrary positive constant. [For convenience, we shall assume that $\Gamma$ is the unit matrix in the following sections.]

Equation (23) implies that adaptation ceases when the augmented error $\varepsilon(t)$ is not greater than $(v_0 + \delta)$. Defining

$$\begin{aligned} \eta(t) &= \varepsilon(t) & |\varepsilon(t)| &> v_0+\delta \\ \text{and} \quad &= 0 & |\varepsilon(t)| &\le v_0+\delta \end{aligned}$$

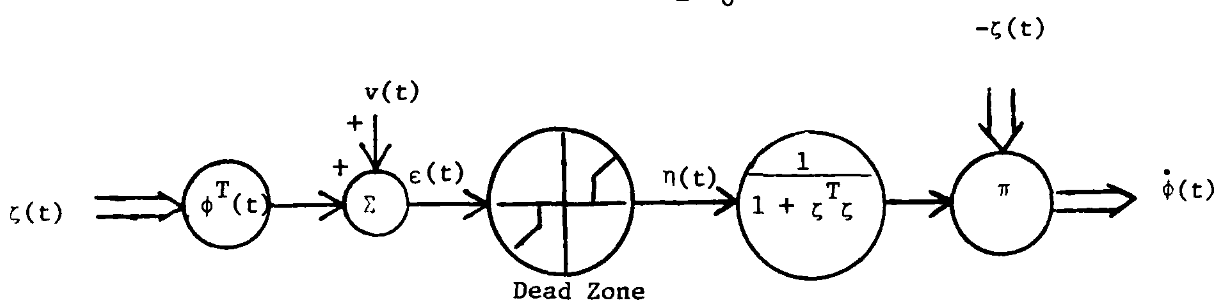



Figure 5

The adaptive law (23) may be stated in terms of $\eta$ as:

$$\dot{\phi} = -\frac{\eta\zeta}{1+\zeta^T\zeta} \tag{24}$$

The adaptive control problem may now be stated in terms of the error equation (21) and the adaptive law (24) as follows:

Given the error equation (21), the adaptive law (24) assures the boundedness of all the parameters and signals of the overall adaptive system.

e) Proof of Stability:

(i) Let $V(\phi) \stackrel{\Delta}{=} \frac{1}{2}\phi^T\phi$. The time derivative of $V$ along a trajectory of the system is given by

$$\dot{V} = \phi^T\dot{\phi} = -\frac{\phi^T\zeta\eta}{1+\zeta^T\zeta} \leq 0 \qquad \forall\ t \in [t_0,\infty). \tag{25}$$

and hence $\phi$ is bounded. Since $V$ is a non-increasing function of time which is bounded below, it follows that $V(t)$ tends to some limit $V_\infty$ as $t \to \infty$ or

$$\lim_{t\to\infty} \|\phi(t)\| = \|\phi(\infty)\| \ .$$

Since $\|\phi(t)\|$ is bounded it follows that all signals within the system can grow at most exponentially. In particular, we have

$$\|\dot{\zeta}\| \leq M_1 \|\zeta\| + M_2 \tag{26}$$

where $M_1$ and $M_2$ are constants. Further, since $\dot{\phi} = -\frac{\eta\zeta}{1+\zeta^T\zeta} = -\frac{\zeta(\phi^T\zeta + v)}{1+\zeta^T\zeta}$ when $|\varepsilon(t)| > v_0 + \delta$ and zero otherwise, $\|\dot{\phi}\|$ is bounded.

(ii) Let $\Omega_1$ and $\Omega_2$ be sets defined by

$\Omega_1 \stackrel{\Delta}{=} \{t \,|\, |\phi^T\zeta + v| \leq v_0 + \delta\}$, $\Omega_2 \stackrel{\Delta}{=} \{t \,|\, |\phi^T\zeta + v| > v_0 + \delta\}$ and $\Omega_1 \cup \Omega_2 = R^+$.

From (25) we have

$$V(\infty) - V(t_0) = \int_{t_0}^{\infty} \dot{V}(\tau)d\tau = \int_{\Omega_2} -\frac{\phi^T\zeta(\phi^T\zeta + v)}{1+\zeta^T\zeta}\,d\tau \tag{27}$$

$$< \infty \qquad \forall\ t_0 \in R^+.$$

if $\phi(t_0)$ and hence $V(t_0)$ are bounded.

From the definition of the dead zone the following inequalities can be derived:

$$\frac{\delta}{v_0 + \delta} \, |\phi^T\zeta + v| \lesssim |\phi^T\zeta| \leq \frac{2v_0 + \delta}{v_0 + \delta} \, |\phi^T\zeta + v| \tag{28}$$

for $t \in \Omega_2$.

From (27) and (28) we have

$$\int_{\Omega_2} \frac{(\phi^T\zeta)^2}{1 + \zeta^T\zeta} \, d\tau < \infty \; ; \int_{\Omega_2} \frac{(\phi^T\zeta + v)^2}{1 + \zeta^T\zeta} \, d\tau < \infty \, . \tag{29}$$

Since $\dot{\phi} \equiv$ for $t \in \Omega_1$

and $\dot{\phi}^T\dot{\phi} = \dfrac{\zeta^T\zeta(\phi^T\zeta + v)^2}{(1 + \zeta^T\zeta)}$ for $t \in \Omega_2$

it follows that $\dot{\phi} \in L^2$. (30)

Let $\mu(\Omega)$ represent the Lebesque measure of $\Omega$. We first consider the case when $\mu(\Omega_2) = \infty$ or the total time during which adaptation takes place is infinite. Since $\|\dot{\phi}\|$ is bounded, by (26) $\|\dot{\zeta}\| \leq M_1 \|\zeta\| + M_2$, $\dfrac{(\phi^T\zeta)^2}{1 + \zeta^T\zeta}$ is uniformly continuous on the open set $\Omega_2$ and from (29) $\displaystyle\int_{\Omega_2} \frac{(\phi^T\zeta)^2}{1 + {}^T} \, d\tau < \infty$, it follows that

$$\lim_{t\to\infty} \frac{\phi^T\zeta}{(1 + \zeta^T\zeta)^{1/2}} = 0 \qquad t \in \Omega_2 \, . \tag{31}$$

or

$$\phi^T\zeta = o[\sup_{\tau \leq t} \|\zeta(\tau)\|] \qquad \tau, t \in \Omega_2 \, . \tag{32}$$

(iii) Since by definition $W_m(s)\omega(t) = \zeta(t)$, and by (30) $\dot{\phi} \in L^2$ it follows from the results derived in [1] that

$$W_m(s)\phi^T\omega = \phi^T\zeta + o[\sup_{\tau \leq t} \|\omega(\tau)\|] \tag{33}$$

From equation (17) the output of the plant $y_p$ can be expressed as

$$y_p = W_m(s)\phi^T\omega + \nu_1 = \phi^T\zeta + o[\sup_{\tau \leq t} \|\omega(\tau)\|] + \nu_1 \tag{34}$$

where $\nu_1$ is a bounded function of time dependent on the reference input r and the disturbance $v_1$ ($\nu_1 = y_m + v$).

The following two cases are of special interest.

Case (a): Let a finite time $t_1$ exist such that $t \in \Omega_1$ for all $t \geq t_1$. In this case the auxiliary error $\phi^T\zeta + v$ lies inside the dead zone and $\dot{\phi} \equiv 0$ for all $t \geq t_1$. Since $\phi^T\zeta$ is bounded it follows from (34) that

$$|y_p(t)| = o[\sup_{\tau \leq t} \|\omega(\tau)\|] + \nu_2(t) \tag{35}$$

where $\nu_2$ is a bounded signal. Equation (35) assures the boundedness of $|y_p(t)|$.

Case (b): Let a constant $t_2$ exist such that $t \in \Omega_2$ for all $t \geq t_2$. In this case the auxiliary error signal lies outside the dead zone for all $t \geq t_2$ and hence adaptation takes place for an infinite interval of time. From (32) and (34) we now have

$$\begin{aligned} |y_p(t)| &= o[\sup_{\tau \leq t} \|\zeta(\tau)\|] + o[\sup_{\tau \leq t} \|\omega(\tau)\|] + \nu_2(t) \\ &= o[\sup_{\tau \leq t} |y_p(\tau)|] + \nu_2(t) \end{aligned} \tag{36}$$

which again assures the boundedness of $|y_p(t)|$.

From cases (a) and (b) we conclude that the signals in the feedback loop cannot grow in an unbounded fashion with the auxiliary signal within the dead zone for all $t \geq t_1$ or outside the dead zone for all $t \geq t_2$ for constants $t_1$ and $t_2$. Hence instability is possible only if the auxiliary error is alternately in $\Omega_1$ and $\Omega_2$. Case (c) considers this general case.

Case (c): $\mu(\Omega_1) = \infty$ $\mu(\Omega_2 = \infty$. Let the output of the plant $y_p(t)$ and hence all the signals in the system including $\omega(t)$ and $\zeta(t)$ grow without bound. From (21),(31) and (32) we have

$$|\phi^T\zeta| < 2v_0 + \delta < \infty \qquad t \in \Omega_1$$

and

$$|\phi^T\zeta| = o[\sup_{\tau \leq t} \|\zeta(\tau)\|] \quad t \in \Omega_2.$$

Hence, from (34)

$$|y_p(t)| = o[\sup_{\tau \leq t} \|\zeta(\tau)\| + o[\sup_{\tau \leq t} \|\omega(\tau)\| ] + \nu_1(t) \qquad (37)$$

Since $\|\zeta(\tau)\|$ and $\|\omega(\tau)\|$ are $O[\sup_{\tau \leq t} |y_p(\tau)|]$ (34) may be expressed as

$$|y_p(t)| = o[\sup_{\tau \leq t} |y_p(\tau)|] + \nu_1(t)$$

which contradicts the assumption that $y_p(t)$ grows without bound.

f) Behavior of the Adaptive System:

In view of the results of section IIIe we conclude that all the signals and the parameters in the adaptive loop are uniformly bounded. Let $\|\zeta\| \leq A$ for some constant A and $t \in \Omega_2$.

$$-\frac{\phi^T\zeta(\phi^T\zeta + v)}{1 + \zeta^T\zeta} \leq -\frac{\delta(v_0 + \delta)}{1 + A^2}$$

or

$$\dot{V} = \phi^T\dot{\phi} \leq -\frac{\delta(v_0 + \delta)}{1 + A^2} < 0 \qquad (38)$$

Equation (38) implies that $\dot{V}$ is strictly less than zero when $t \in \Omega_2$ and adaptation takes place. Since V is a non-negative function, adaptation can take place only for a finite interval of time $T \leq \frac{\|\phi(0)\|^2(1 + A^2)}{\delta(v_0 + \delta)}$ or $\mu(\Omega_2) < \infty$ and $\mu(\Omega\ ) = \infty$. In other words, except for a set $\Omega_2$ of finite measure the system is linear time-invariant.

IV. Choice of the Dead Zone:

From the preceding analysis it is clear that the proper choice of the size of the dead zone i.e. $v_0 + \delta$ is crucial for establishing global stability in the presence of an external disturbance. As mentioned in section IIId, this is dependent not only on the bound on the disturbance but also on the unknown plant parameters. In view of its importance, we discuss briefly in this section some of the many considerations that enter into the choice of $v_0 + \delta$. As pointed out in section II, a larger dead zone implies a shorter period of time during which adaptation takes

place and in general larger output and parameter errors. Since $\delta$ is an arbitrary positive constant, the following discussion will deal entirely with the choice of $v_0$.

The signal v(t) which appears in the error equation (18) corresponds to the measured output error when $\theta(t) \equiv \theta^*$. Hence

$$\begin{aligned} v(t) &= [1 + W_m(s)\{\theta_{(2)}^{*T}(sI-F)^{-1}g + \theta_{2n-1}^*\}]v_1(t) \\ &= \frac{p_1(s)}{q_1(s)} \; v_1(t) \end{aligned} \tag{39}$$

where $p_1(s)$ and $q_1(s)$ are monic polynomials of degree $(n^* + n-1)$, $q_1(s)$ is Hurwitz and the coefficients of the first $n^*$ terms of $p_1$ are the same as those of $q_1$. $p_1$ and $q_1$ can be expressed more explicitly in terms of the model and plant characteristic polynomials $D_p(s)$ and $D_m(s)$ as follows:

$$\frac{p_1(s)}{q_1(s)} = \frac{D_p(s)\Lambda_1(s)}{D_m(s)\Lambda(s)} \tag{40}$$

where $\Lambda(s) = |(sI-F)|$ and $\Lambda_1(s)$ *is a monic polynomial of degree* $(n^*-1)$ such that the first $n^*$ terms of $D_p(s)\Lambda_1(s)$ match those of $D_m(s)\Lambda(s)$.

The prior knowledge needed to calculate the size of the dead zone is the bound $v_0$ of v(t). From (39) and (40) it follows that bounds on $v_1(t)$ as well as the coefficients of $D_p(s)$ are adequate to calculate $v_0$. If a bound on the norm of the parameter vector $\begin{bmatrix} \theta^*_{(2)} \\ \theta^*_{2n-1} \end{bmatrix}$ is known $v_0$ can also be computed from equation (39). In all cases it is worth noting that no prior knowledge of the zeros of the plant transfer function are needed to determine $v_0$.

Simulation results indicate that the sizes of the dead zone computed using the above methods generally tend to be too conservative. Hence equations (39) and (40) are primarily of theoretical interest to assure that a finite $v_0$ exists. In practice, since the plant together with the controller is stable, the plant coefficients can be estimated on-line and in turn used to determine less conservative values of $v_0$.

Systematic procedures for accomplishing this in a stable fashion are under investigation.

## V. Simulation:

Example 2: The adaptive control of a second order plant in the presence of an external disturbance was simulated under a variety of conditions. The plant, model and filter transfer functions are given by:

$$W_p(s) = \frac{1}{s^2-1} \quad ; \quad W_m(s) = \frac{1}{s^2 + 4s + 3} \quad \text{and } 1/\Lambda(s) = 1/(s + 2).$$

In this case $\Lambda_1(s) = s + 6$ and if the disturbance is $v_1(t) = 4 \sin t$, $v_0$ can be computed as 4.87.

(i) Figure 6a shows the output error as well as controller parameters with no dead zone in the adaptive law and no reference input. While the output error tends to zero the parameter error is seen to increase without bound.

(ii) Figure 6b shows the same system (i.e. without dead zone) when a sufficiently rich reference input is used. In this case all signals and parameters are seen to be bounded.

(iii) In Figures 6c and d, the reference input is identically zero but a dead zone is used. In both cases the parameters are bounded and reach constant values in a finite period of time. Figure 6c reveals that a smaller dead zone of $v_0 + \delta = 2$ as compared to 5 for Figure 6d results as expected in a smaller output error.

(iv) Figure 6e with a sufficiently rich input and a dead zone in the adaptive law (same as in c) results in a smaller output error.

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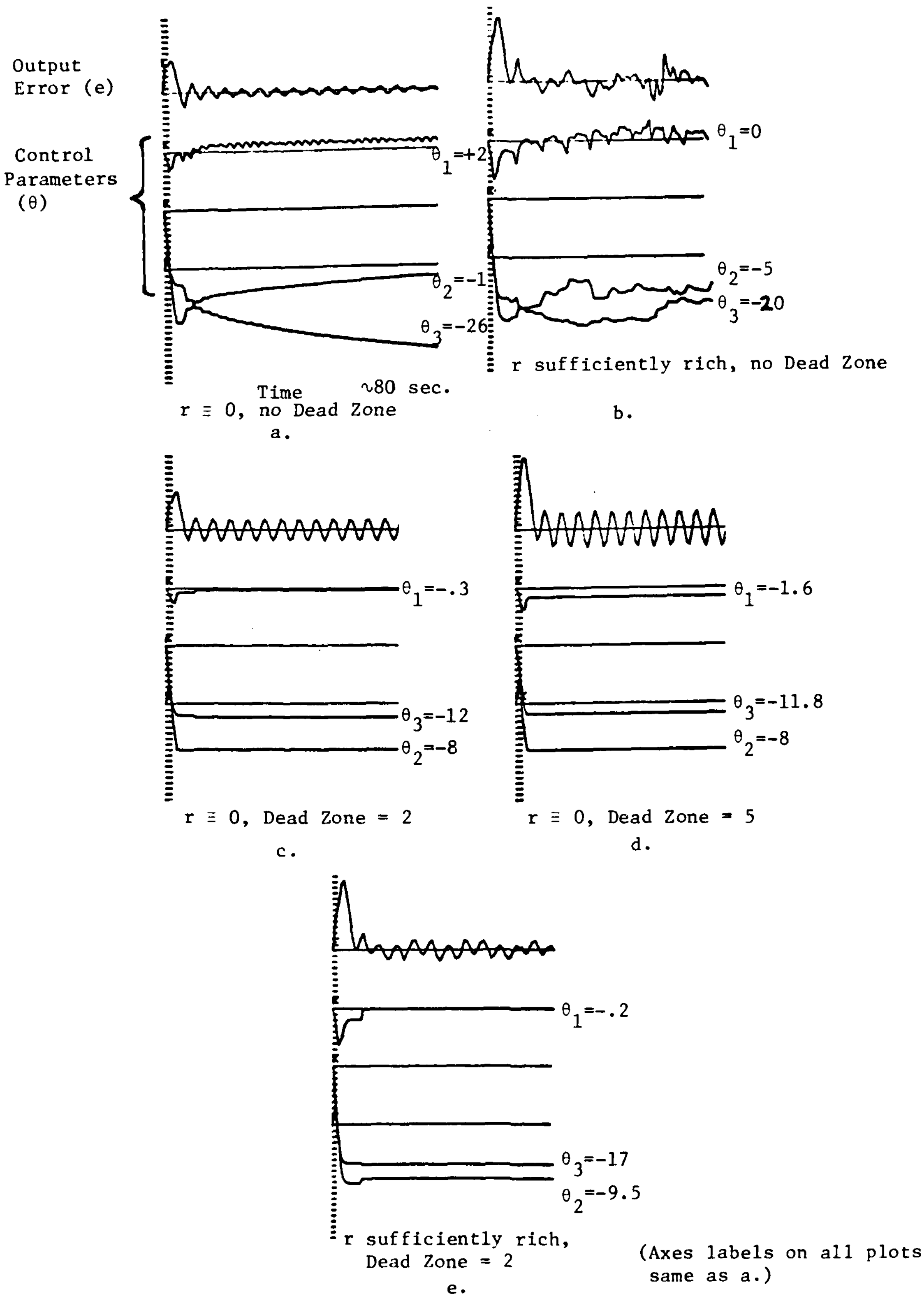

Output Error (e)
Control Parameters (θ)
θ1=+2
θ2=-1
θ3=-26
Time ∿80 sec.
r ≡ 0, no Dead Zone
a.
θ1=0
θ2=-5
θ3=-20
r sufficiently rich, no Dead Zone
b.
θ1=-.3
θ3=-12
θ2=-8
r ≡ 0, Dead Zone = 2
c.
θ1=-1.6
θ3=-11.8
θ2=-8
r ≡ 0, Dead Zone = 5
d.
θ1=-.2
θ3=-17
θ2=-9.5
r sufficiently rich, Dead Zone = 2
e.
(Axes labels on all plots same as a.)


Figure 6

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